A simple method for estimating MSY from catch and resilience

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Abstract
The Law of the Sea requires that fish stocks are maintained at levels that can produce the maximum sustainable yield (MSY). However, for most fish stocks, no estimates of MSY are currently available. Here, we present a new method for estimating MSY from catch data, resilience of the respective species, and simple assumptions about relative stock sizes at the first and final year of the catch data time series. We compare our results with 146 MSY estimates derived from full stock assessments and find excellent agreement. We present principles for fisheries management of data-poor stocks, based only on information about catches and MSY.

Keywords Carrying capacity, data-poor stocks, harvest control rules, intrinsic rate of population increase, maximum sustainable yield

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Introduction

The need for simple methods

In the Law of the Sea of 1982 (UNCLOS 1982), which entered into force in 1994, the nations of the World have agreed to maintain exploited populations of marine organisms at levels that can produce the maximum sustainable yield MSY. Respective management systems have been introduced by Australia (DAFF 2007), New Zealand (MFNZ 2008), and the USA (MSA 2006), and Europe plans an implementation by 2013 (EC 2011). However, for the vast majority of exploited populations or stocks, no estimates of MSY are available. Thus, there is a need for simple methods that allow inclusion of such stocks in MSY management schemes.

Outline of the Catch-MSY method

The simplest model-based methods for estimating MSY are production models such as the Schaefer (1954). At a minimum, these models require time series data of abundance and removals to estimate two model parameters: the carrying capacity $k$ and the maximum rate of population increase $r$ for a given stock in a given ecosystem. While estimates of removals (defined here as catch plus dead discards) are available for most stocks, abundance estimates are difficult and costly to obtain and are mostly missing. However, given only a time series of removals, a surprisingly narrow range of $r$-$k$ combinations is able to maintain the population such that it neither collapses nor exceeds the assumed carrying capacity. This set of viable $r$-$k$ combinations can be used to approximate MSY. Here, we present a simple method that uses catch data plus readily available additional information to approximate MSY with error margins. We demonstrate the application for two stocks, Greenland halibut ($Reinhardtius hippoglossoides$, Pleuronectidae) and Strait of Georgia lingcod ($Ophiodon elongatus$, Hexagrammidae). We apply the method to 48 stocks of the Northeast Atlantic for which independent MSY estimates were available from a previous study and compare the results. We also apply the method to 98 global stocks with MSY estimates. Finally, after pointing out some caveats, we propose simple management rules based on catch and MSY.

Material and methods

Model and assumptions

The Catch-MSY method as proposed here was inspired by the stock reduction analysis of Kimura and Tagart (1982) and Kimura et al. (1984). As input data, it requires a time series of removals, prior ranges of $r$ and $k$, and possible ranges of relative stock sizes in the first and final years of the time series. It then uses the Schaefer production model to calculate annual biomasses for a given set of $r$ and $k$ parameters. As no prior distributions of $r$ and $k$ are available for most fish stocks, we randomly draw $r$-$k$ pairs from a uniform prior distribution and then use a Bernoulli distribution as the likelihood function for accepting each $r$-$k$ pair that has never collapsed the stock or exceeded carrying capacity and that results in a final relative biomass estimate that falls within the assumed range of depletion. Additional process errors can also be added to the model if desired. Absent process errors, as in our examples, are equivalent to assuming an observation error only model that is deterministic. A detailed description of the parameters and equations is given in the Appendix. The R-code for batch processing of the 146 stocks and the catch data are available from http://www.fishbase.de/rfroese/ with file names of CatchMSY_2.r, RAM_MSY.csv, and ICESct2.csv concatenated to the URL, respectively.

Data sources

We used assessment data for 48 stocks of 19 species of the Northeast Atlantic, as available in the ICES Stock Summary Database downloaded from www.ices.dk/datacentre/StdGraphDB.asp in September 2011. We extracted estimates of $F_{0.1}$ from ICES advice documents for 2011, as available from www.ices.dk. We also used the estimates of $F_{msy}$, MSY, and carrying capacity $k$ for these stocks from Froese and Proelss (2010). For each species, we got a resilience classification from FishBase (Froese and Pauly 2011). These stocks spanned a wide range of sizes and exploitation rates, ranging in spawning stock biomass from 1000 tonnes to 12 million tonnes, with exploitation rates $F/F_{msy}$ of 0.5 to 5.8. The advantage of this data set was the application of the same standard methods across all stocks, and the provision of MSY with 95% confidence limits by Froese and Proelss (2010).
The disadvantage of this data set was that, with one exception, it only contained species with medium resilience. We, therefore, also used working group assessments of MSY for 98 stocks from the RAM legacy database (Ricard et al. 2011). For the batch analysis of these stocks, we derived default ranges of relative biomass in the first and final year of the time series, based on respective catches relative to the maximum catch (Froese et al. 2012), see Table 1.

Random samples of the carrying capacity parameter \( (k) \) were drawn from a uniform distribution where the lower and upper limits were given by the maximum catch in the time series and 100 times maximum catch, respectively. Note that such upper bound for \( k \) means that catches never exceeded 1% of the carrying capacity. If this were indeed the case, catches would contain very little information about the productivity of the stock and the Catch-MSY method should not be applied. Given their nearly unexploited status, such stocks are not in immediate need of management.

We used resilience estimates from FishBase, which are based on Musick (1999) as modified by Froese et al. (2000), to assign default values to the allowed range for the random samples of the maximum intrinsic rate of population increase \( r \) (Table 2). Note that we do not propose application of the Catch-MSY method with the default values in Tables 1 and 2 for serious stock assessment. Rather, we would expect that the best available knowledge about the respective stocks is used.

As most probable values from the resulting density distributions, we used the geometric means of \( r, k \), and MSY, where MSY was calculated from the \( r-k \) pairs (see Appendix). We chose geometric mean instead of mean, median, or mode because it was the only estimate where the central MSY value derived after the calculation of MSY for each \( r-k \) pair was about the same as the one derived by using the respective central values of \( r \) and \( k \). For example, for Western Baltic cod, median MSY calculated from \( r-k \) pairs was 38 335 tonnes, whereas MSY calculated from median \( r \) and \( k \) was 38 997 tonnes, a difference of 662 tonnes. For the geometric mean, the respective values were 38 975 and 38 906, a difference of only 69 tonnes. Thus, the geometric mean seemed to better capture the distributions of \( r \), \( k \), and MSY.

As a measure of uncertainty, we used two times the standard deviation of the logarithmic mean. This implies that, with a roughly lognormal distribution, about 95% of the MSY estimates would fall within this range.

## Results and discussion

### Applying the Catch-MSY method to Greenland halibut

Fig. 1 shows the graphical output of the Catch-MSY method as applied to the Greenland halibut, a species with low resilience (Froese and Pauly 2011; see Table 1). Panel A shows the time series of catches with overlaid estimate of MSY = 24 900 tonnes and the limits (19 800–31 400) that contain about 95% of the MSY estimates derived from the \( r-k \) pairs. This is not significantly different from an independent estimate for this stock of MSY = 31 023 tonnes with 95% confidence limits of 19 800–31 400 tonnes (Froese and Proelss 2010). Panel B spans the prior uniform distribution of \( r = 0.05–0.5 \) and \( k = 89 484–8 948 \) 400 tonnes. The \( r-k \) combinations (1st iterations) that are compatible with the time series of catches occupy only a small corner of that space, showing the typical decline of viable \( r-k \) pairs with increasing \( r \). Panel C is a magnification of the \( r-k \) pairs (after 2nd iterations with new upper limit for \( k \)) in log space, with overlaid lines indicating the \( r-k \) combinations that would result...
Panels D to F show the posterior densities of $r$, $k$, and MSY, respectively. Applying the Catch-MSY method to other stocks with available MSY estimates

The key question obviously is how well the MSY estimates derived with the Catch-MSY method compare with a wide range of MSY estimates from full stock assessments. For this comparison, we used 48 stocks from the Northeast Atlantic and 98 stocks from all over the world, analyzed with the default assumptions as described above. These default settings found $r$-$k$ pairs for all Northeast Atlantic stocks and for most of the global stocks. Fig. 2 shows a comparison of the respective MSY estimates for the 48 Northeast Atlantic stocks. A log–log linear regression accounted for 98.6% of the variability of Catch-MSY estimates relative to full assessment estimates of MSY, with an intercept not significantly different from the origin ($n = 48$, log intercept = $-0.05$, 95% CL = $-0.118$–$-0.018$) and a slope not significantly different from 1 (slope = 1.003, CL = 0.967–1.039, $r^2 = 0.986$). The 95% confidence limits of MSY provided by Froese and Proelss (2010) overlapped in 42 of the 48 stocks with the double standard deviation used as an error margin by the Catch-MSY method, suggesting that these MSY estimates were not significantly different.

For the global stocks, the default settings did not result in suitable $r$-$k$ combinations for about 10 of the 98 stocks, mostly because these stocks had intermediate resilience (between very low and low or between low and medium, see Table 2), or because they were very lightly exploited, with maximum catches of 2–30% of the MSY estimate of the respective working groups, see outliers in Fig. 3. As pointed out previously, in very lightly fished stocks, the time series of catches does not contain sufficient information about productivity, and the Catch-MSY method should not be applied. But overall, most of the Catch-MSY estimates for the global stocks fell within a range of 0.5–1.5 of the independent estimate, see respective lines in
Fig. 3. Thus, the Catch-MSY method appears well suited to provide preliminary approximations of MSY in cases where abundance data are lacking.

How good are the estimates of $r$ and $k$?

A stock that was able to produce the cumulative historical catch must have had a certain productivity, for which the maximum sustainable yield is an appropriate measure. MSY is a function of $r$ and $k$, which in the Schaefer model have a log linear negative correlation with a slope of $-1$ (Fig. 1c). In other words, the observed catches may have been produced by a small population with high $r$ or a large population with small $r$, or a very lightly exploited population with any combination of $r$ and $k$. In the last case, catch data are insufficient to estimate population properties, error margins for $r$, $k$, and MSY will be very wide, and MSY will be underestimated (outliers in Fig. 3). High values of $r$ cause strong fluctuations in stock size, with the associated risks of over-shooting carrying capacity or going extinct (May 1974, 1976). Also, the larger $k$ is relative to catches, the wider is the range of $r$ values that allow the population to sustain those catches. These two effects may explain the typical triangular shape of viable $r$-$k$ pairs in log $r$-$k$ space (Fig. 1c), with few pairs at high and most pairs at lower $r$ values. This triangle only expands by its short side, when the range for prior $r$ is reduced and for prior $k$ increased.

The triangular shape of viable $r$-$k$ space does not affect much the estimation of a representative value for MSY, as the line of $r$-$k$ pairs giving the same MSY is anchored near the center of the triangle (Fig. 1c). However, the estimates of most probable central values for $r$ and $k$ are strongly dependent on the lower limit chosen for $r$ and the upper limit chosen for $k$. While the lower limit for
is assumed to represent the best available prior knowledge, the upper limit for \( k \) was chosen arbitrarily as 100-fold the maximum catch in the time series. We used the following method for finding an upper limit of \( k \) better corresponding with the lower limit of \( r \), based on the knowledge gained in a first analysis of the data: We selected as upper limit of \( k \) the smallest viable \( k \) value at the lower limit of \( r \). This provided a clear new upper cut off for \( k \) determined by the prior lower limit of \( r \) (Fig. 1c).

In Figs 4–6, we compare the \( r \) and \( k \) estimates of the Catch-MSY method with related fisheries reference points. Figure 4 shows a plot of \( k \) over unexploited total biomass from Froese and Proelss (2010). The points scatter around the 1:1 line, but with an upward bias of about 10%, that is, the Catch-MSY method overestimated carrying capacity and related biomass reference points by about that amount. Similarly, in Fig. 5, most estimates of \( r \) fall below the 1:2 line of the relationship between \( r \) and the fishing mortality \( F_{msy} \) that would result in the biomass that can produce maximum sustainable yield (data from Froese and Proelss 2010). A better match is obtained in Fig. 6, where \( r \) is plotted over the conservative fishing mortality \( F_{0.1} \) derived by ICES working groups from yield per recruit analysis (Cadima 2003). The rectangular distribution of the data points in Figs 5 and 6 stems from the fact that most species were of medium resilience and thus had the same default lower prior limit of \( r = 0.2 \).

In summary, while MSY estimates of the Catch-MSY method are fairly robust with regard to initial assumptions and in very good agreement with estimates derived with more demanding methods, the \( r \) and \( k \) estimates strongly depend on the lower prior limit for \( r \) which thus must be carefully set.

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**Figure 4** Plot of carrying capacity \( k \) estimated by the Catch-MSY method over estimates of unexploited total biomass from Froese and Proelss (2010). Note that the majority of points fall above the 1:1 line (dotted), that is, the Catch-MSY method tends to overestimate \( k \) and reference points derived from it.

**Figure 5** Plot of maximum intrinsic rate of population increase \( r \) over estimates of fishing pressure associated with maximum sustainable yields \( F_{msy} \), as obtained from Froese and Proelss (2010). Note that most estimates fall below the expected 1:2 line (dotted), that is, the Catch-MSY method tends to underestimate \( r \) and derived reference points such as \( F_{msy} \).
From a management point of view, the bias of $r$ and $k$ is precautionary, because it suggests higher thresholds for biomass and lower thresholds for fishing mortality.

Applying the Catch-MSY method to the 'data-poor' Strait of Georgia lingcod

To demonstrate this simple method for estimating MSY from catch time series data, we used the historical landings from the Strait of Georgia lingcod fishery, for which no abundance data were available (Fig. 7) (King 2001). Landings of lingcod from this region date back to 1889. The commercial fishery largely consisted of a handline fishery; however, lingcod were also taken in trawl fisheries, starting in the 1940s. The stock was considered depleted and the commercial fishery was closed in 1990. The remaining recreational fishery was closed in 2002 (Logan et al. 2005).

The lingcod fishery began around 1860 (King 2001), and thus, we assumed that the biomass at the start of the time series in 1889 was already below carrying capacity. As early catches were modest, we assumed a start biomass of 0.8 $k$. For the prior densities of $r$, we took the uniform distribution for species with very low resilience with $r = 0.015-0.1$ (Table 1). Given the known depleted state of the stock, we assumed that relative biomass in 2002 was between 1% and 25% of the carrying capacity. Inserting the estimated MSY in Fig. 7 facilitated the interpretation: Landings reached the lower range of MSY estimates in the 1900s, exceeded MSY in the 1910s and the upper range of MSY estimates between 1920 and 1960, and declined thereafter. Despite the closure of the commercial fishery in 1990, the stock showed no consistent signs of recovery in 2000 (Logan et al. 2005). A full assessment is still not available for this stock (DFO 2012), but Fig. 7 suggests that...
the massive and prolonged overshooting of MSY was the cause of depletion in 1990.

Caveats of the Catch-MSY approach

A key assumption in the Catch-MSY approach as laid out here is the ability to define a reasonable prior range for the parameters of the Schaefer model. In our case studies, we have arbitrarily chosen 100 times the maximum catch as the upper bound for $k$. In a developing fishery, or a fishery that has a continuous increase in catch, it will be more difficult to define the upper bound of $k$ because the maximum potential has yet to be realized. Another key assumption is the stated range of depletion for which to accept or reject the sets of $r$-$k$ pairs of parameters. The lower depletion limit defines the lower boundary of the resulting MSY distribution, and the upper depletion limit and the range of values for $k$ determine the upper bound of MSY. To be clear, these depletion levels are assumptions about the current state of the stock. Finally, the Catch-MSY approach also assumes a stationary production function or in this case, no change in the parameters of the Schaefer model over time.

Other methods for estimating MSY from catch data

MacCall (2009) provides estimates of depletion-corrected average catch (DCAC) based on catch data and estimates of natural mortality or of the depletion in stock size caused by fishing. As MacCall (2009) points out, this method does not give estimates of MSY but suggests rather a ‘...moderately high yield that is likely to be sustainable, while having a low probability that the estimated yield level exceeds MSY...’ Dick and MacCall (2011) present a depletion-corrected stock reduction analysis (DB-SRA) based on a time series of annual catches, the rate of natural mortality $M$, the $F_{nys}/M$ ratio, the age at maturity, the $B_{nys}/k$ ratio, and an estimate of relative biomass near the end of the time series. The method then applies a production function and accepts estimates of $k$ from biomass trajectories that never became negative. Outputs of the model are MSY and unexploited biomass $k$. The authors tested their method on 31 northeast Pacific groundfish stocks off the US West coast and conclude that ‘...for most stocks we evaluated, median estimates of MSY and $K$ [...] tend to be between one-half and double the assessment value’.

The DB-SRA method is similar to the Catch-MSY method we present here. However, the application of the DB-SRA method requires more knowledge about the respective stock, such as a good estimate of productivity. In comparison, the Catch-MSY method requires as input only fairly wide ranges of potential productivity, which may be derived from resilience estimates, and fairly wide ranges of initial and final relative abundance, which may be derived from initial and final catches relative to the maximum catch in the time series (Froese et al. 2012).

Wetzel and Punt (2011) evaluate the performance of the DCAC and DB-SRA methods and conclude that both are reasonably robust with regard to errors in natural mortality (used for estimating productivity) but sensitive to overly optimistic depletion ratios.

Principles of MSY-based management

We have shown above that with catch data and simple assumptions about resilience and the status of the stock, reasonable estimates of MSY with error margin can be obtained. But what are harvest control rules for management based only on catch and MSY? An obvious first rule is that catches shall never exceed MSY and that the lower margin of error of MSY should be used as target for total allowable catch if stock size can be assumed to be above 0.5 $k$. Note that this lower margin will be further decreased if process error is included in the model. Stock size is likely to be below 0.5 $k$ if any of the following observations is true:

1. Catches in the past have exceeded MSY;
2. Catch per unit of effort (CPUE) is stable or decreasing, instead of slightly increasing, as can be expected as result of ‘effort creep’ (Marchal et al. 2007);
3. Mean length in the catch has declined and the proportion of large fish is less than 30-40% of that known from the beginning of the fishery (Froese 2004).

If an overfished status of the stock cannot be excluded, catches should be reduced strongly until increases in CPUE and increases of maximum length in the catch indicate a recovery. Catches can then be increased slowly, that is, slower than
the expected increase in biomass, until the lower error margin of MSY is reached.

We hope that this simple method and the proposed principles will prove useful in the management of data-poor stocks.

Acknowledgement

We thank Daniel Pauly for constructive criticism and for reminding us not to destroy ‘the beautiful simplicity of the thing’ by too many additional assumptions. We also thank Selina Heppell for inviting us to a workshop that inspired this idea. Steven Martell acknowledges financial support from NSERC Discovery Grant. Rainer Froese acknowledges support by the Future Ocean Excellence Cluster 80, funded by the German Research Foundation on behalf of the German Federal State and State Governments. He also acknowledges support from the European Union’s Seventh Framework Programme (FP7/2007-2013) under grant agreement no. 244706/ECOKNOWS project. However, the paper does not necessarily reflect the views of the European Commission (EC) and in no way anticipates the Commission’s future policy in the area.

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Ricard, D., Minto, C., Jensen, O.P. and Baum, J.K. (2011) Examining the knowledge base and status of commercially exploited marine species with the RAM Legacy
Appendix 1

The method outline here for approximating MSY is based on a very simple Schaefer production model, and it should be noted here that other models with alternative assumptions about the form of the stock productivity relationship could be substituted with the additional structural assumptions. The primary objectives of this method are (i) to devise a very simple method that can be applied to any catch time series, (ii) the method must be easy to understand and implement so that it can be used by many people involved in fisheries science and management, and (iii) the method requires few additional assumptions.

The minimum data requirement is a catch time series from a specific area that is normally defined as a unit stock where the population is closed to immigration and emigration (Table A1, equation 1). In addition to the catch data, a range of initial and current depletion levels (i.e., the current stock size relative to the unfished carrying capacity and current depletion levels) must also be specified; these are denoted by \( \lambda_{01} \) and \( \lambda_{02} \) for the initial stock size and by \( \lambda_{1} \) and \( \lambda_{2} \) for the final lower and upper limits, respectively. The last remaining assumption is to specify the standard deviation in the process errors \( \sigma_{v} \); process errors are assumed lognormal, independent, and identically distributed (10). If \( \sigma_{v} = 0 \) , this is equivalent to assuming a deterministic model. The model parameters (4) of interests are the carrying capacity \( k \) and the maximum intrinsic rate of population growth \( r \). Starting with an assumed relative biomass of \( B1 = \lambda_{01} \) in the first year, biomass in subsequent years is calculated based on (6), where the observed catch is subtracted from the start of the year biomass. This assumes the catch is measured without error, unless \( \sigma_{v} > 0 \). This is repeated for additional initial relative biomasses, in steps of 0.05 between \( \lambda_{01} \) and \( \lambda_{02} \).

A very simple importance sampling procedure is then used to map the joint distribution of model parameters (in this case, \( r \) and \( k \) of the Schaefer production model) that lead to current depletion levels between \( \lambda_{1} \) and \( \lambda_{2} \). In cases where combinations of \( (r, k) \) lead to the population going extinct or overshooting \( k \) before the end of the time series, we simply assign 0 for that parameter combination. For combinations of \( (r, k) \) that result in final stock sizes between \( \lambda_{1} \) and \( \lambda_{2} \), we assign a value of 1 (equation 7). Then, for each parameter combination that results in a viable population at the end of the time series, estimates of MSY can be calculated from the population parameters (11).

The basic algorithm is implemented as follows:

1. Specify the initial status of the stock (\( \lambda_{01} \) and \( \lambda_{02} \)) and lower (\( \lambda_{1} \)) and upper (\( \lambda_{2} \)) limits of the final status of the stock (e.g., values of \( \lambda_{01} = 0.5 \) and \( \lambda_{02} = 0.9 \) imply that the stock was between half and 90% of carrying capacity at the beginning of the time series, and \( \lambda_{1} = 0 \) and \( \lambda_{2} = 1 \) imply that the stock is somewhere between completely depleted and at its carrying capacity at the end). Also, specify \( \sigma_{v} \) to a value greater than 0 if you wish to include a stochastic component.
2. Draw a trial parameter set \( \Theta_{i} \) from the respective prior distributions (e.g., equations 8, 9, and 10).
3. Initialize the population model at the trial value of \( k_{i} \) (5).
4. Update the biomass next year using the Schaefer production model (6).
5. Calculate the likelihood of the parameter vector \( \Theta_{i} \) using (7).
6. Repeat steps 2–5 many times (e.g., 100 000) and store the 0 or 1 likelihood for each trial.
7. Plot distributions of management quantities (11) only for cases in which the likelihood is 1.


Table A1 A simple Schaefer production model and the corresponding management parameters.

<table>
<thead>
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<tr>
<td>( c ) observed catch from ( t = 1 ) to ( t = n ) years</td>
<td>(1)</td>
</tr>
<tr>
<td>( \lambda_1, \lambda_2 ) lower and upper bounds for relative biomass in year 1</td>
<td>(2)</td>
</tr>
<tr>
<td>( \delta_1, \delta_2 ) lower and upper bounds for depletion level</td>
<td>(3)</td>
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<tr>
<td>( \sigma_r ) process error standard deviation</td>
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<table>
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<td>( B_t = \lambda_0 k ) ( \exp(r t) )</td>
<td>(5)</td>
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<tr>
<td>( B_{t+1} = [B_t + r B_t (1 - B/k) - c_t] \exp(r t) )</td>
<td>(6)</td>
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<td>( l(c</td>
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<td>( - 0 ) ( \lambda_1 &gt; B_{t+1}/k &gt; \lambda_2 )</td>
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<tr>
<td>( p(\log(r)) \sim \text{uniform}(\log(l), \log(u)) )</td>
<td>(9)</td>
</tr>
<tr>
<td>( p(\cdot</td>
<td>\theta) \sim \text{normal}(0, \sigma^2) )</td>
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<td>(11)</td>
</tr>
<tr>
<td>( B_{msy} \sim \frac{1}{2} k )</td>
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<tr>
<td>( F_{msy} \sim \frac{1}{2} f )</td>
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